PRINCIPLE OF MATHEMATICAL (WEAK) INDUCTION

Let P(n) be a predicate that is defined for natural numbers n.
Let a be a fixed natural number.

Suppose the following 2 statements are true:

1. Basic Step: P(a) is true
2. Inductive Step: $\forall k \in \mathbb{N}$ s.t. $k \geq a$ $P(k) \Rightarrow P(k+1)$

Then the following statement is true: $\forall n \in \mathbb{N}$ s.t. $n \geq a$, $P(n)$

The supposition that P(k) is true in the inductive step is called the inductive hypothesis (IH)

PRINCIPLE OF STRONG INDUCTION

Let P(n) be a predicate defined for natural numbers n.
Let a and b be fixed natural numbers s.t. $a \leq b$.

Suppose the following 2 statements are true:

1. Basic Step: P(a), P(a+1), ... P(b) are true
2. Inductive Step: $\forall k \in \mathbb{N}$ s.t. $k \geq b$
   $[\forall m \in \{a, a+1, ..., k\}\ P(m))] \Rightarrow P(k+1)$

Then the following statement is true: $\forall n \in \mathbb{N}$ s.t. $n \geq a$, $P(n)$

The supposition that $[\forall m \in \{a, \ldots, k\}\ P(m)]$ is true is called the inductive hypothesis (IH)

Note: Weak induction is a subcase of strong induction with a = b and a weaker inductive hypothesis

RELATIONSHIP BETWEEN RECURSION AND INDUCTION

Recursion	Induction
Initial conditions specify the values $a_0, a_1, ..., a_{i-1}$ for a fixed natural i.	Basic step: prove that property is true for $a_0, a_1, ..., a_{i-1}$ for a fixed natural i.
Recurrence relation specifies a_k with respect to certain of its predecessors $a_{k-1}, a_{k-2}, ..., a_{k-i}$,	Inductive step: prove that if property is true for $a_0, a_1, ..., a_{k-1}$ then it is true for a_k